Lab 3 Test for multifactor pricing models

* The lab report should present your performance of each task in the lab.
* Format: a pdf- le, Times New Roman, 12-point font size, one and half line spaced. The title page must contain your group number, names and personal numbers..

Your need to perform the lab tasks in Excel. In the assignment, you should

* elaborate the estimation approach briefly;
* report and interpret the estimated models and statistical tests but not include all the detailed calculations

The Excel le Lab3.xlsx contains monthly data of

* realized returns of five stocks (used as test assets)
* the risk-free return,
* realized return of a broad stock index (the market portfolio), the small- minus-big (SMB) factor and the high-minus-low(HML) factor.
* three macroeconomic variables: industrial production growth, interest rate spread (10 year US government bond rate - FED rate), and oil price returns.

1 Tasks

1. Use the Likelihood ratio approach to test the Fama-French three factor model. Calculate the risk premia of the factors. Test whether the risk premium of each factor is significant.

Instructions:

Likelihood ratio test: estimate both the unrestricted and the restricted models with maximum likelihood method using initial values for alphas and betas.

* + Unrestricted model

***Z****t* = ***α*** + ***β****mZmt* + ***β****SMBSMBt* + ***β****HMLHMLt* + ***s****t* (1.1) E[***s****t*] = 0

E[***s****t****s****jt*] = **Σ**

***Z****t* is an *N ×* 1 vector of excess returns for *N* stocks.

1. Start by calculating excess returns of the ve stocks and that of the market portfolio, *Zt* and *Zmt*. Note that the risk-free rate cancels o after subtracting the return of one portfolio from the other when constructing SMB and HML. So you shouldn't subtract risk free return from SMB and HML.
2. De ning initial values for the parameters. Since *N* = 5 *K* = 3, You need ve initial values for *α*:s and 15 initial values for *β*:s. Initial values are given in Lab3.xlxs.
3. 5 new columns for the residuals by referring to the initial values as

*eit* = *Zit−αi−βimZmt−βi,SMBSMBt−βi,HMLHMLt, i* = 1*,* 2*, . . .* 5

(1.2)

1. Calculate the residual covariance matrix **Σ**: rst select a 5 *×* 5 area and then use the function:

"MMULT(Transpose(e);e)/*T* ", then press CTRL+SHIFT and ENTER.

Note that *T* is the number of observations (198) and ***e*** stands for

the *T × N* matrix of all the residuals.

1. Calculate the inverse of **Σ**, **Σ***−*1: rst select a 5 *×* 5 area and then use the function

MINVERSE(*·*)

1. Calculate ***e****jt***Σ***−*1***e****t* at each time *t* as MMULT(*et*; MMULT(**Σ***−*1; Transpose(*et*))).

*et* is the row of the residual at time t not the entire residual matrix. Since our *et* is a 1 *×* 5 vector and not 5 *×* 1, we transpose *et* after **Σ***−*1 not before **Σ***−*1.

1. Calculate the log likelihood value

ln *L*(***α****,* ***β****,* ***,*** Σ) = *−NT* ln(2*π*) *− T* ln *|***Σ***| −* 1 Σ ***e****j* **Σ***−*1***e*** (1.3)

*t*=1

*T*

2

2

2

*t*

*t*

The function in excel for *π* is pi(); the function for matrix determinant is MDETERM(*·*); the function for ln is LN(*·*).

1. Use solver to maximize this sum by changing the initial values.

Note: do not select "Make Unconstrained Variables Non- Negative".

The procedure here gives you the optimal log likelihood for the unrestricted model ln *L*

* + Restricted model

Now you need to obtain the log likelihood value for the restricted model

***Z****t* = ***β****mZmt* + ***β****SMBSMBt* + ***β****HMLHMLt* + ***s****t* (1.4)

*H*0 : ***α*** = **0**; *H*1 : ***α*** *ƒ*= **0***.*

To avoid repeating all the steps (a) to (f) we can just copy the entire worksheet from the unrestricted model. Then

(a)Set the initial values for the parameters back to the starting values. (b)Use solver to maximize the sum of the loglikelihood function by

changing only the initial values for *β*:s. This keeps the values for *α*:s

at zero, which is the restriction we want to impose in our restricted model.

The procedure here gives you the optimal log likelihood for the restricted model ln *L∗*.

* + The null hypothesis can be tested using the likelihood ratio:

*LR* = *−*2(ln *L∗ −* ln *L*) *∼ χ*2

*N*

(1.5)

Use the function "CHIDIST(*LR*, *N* )" to compute the p-value for the test statistics.

* + The factors are traded portfolios. Note that the risk-free rate cancels o after subtracting the return of one portfolio from the other when constructing SMB and HML. So, the factor risk premia are calculated as the sample averages of the excess market return and the factor

realizations of SMB and HML:

*T*

Σ

*λ*ˆ*m*

= 1 *Z*

*T mt*

*t*=1 *T*

Σ

(1.6)

*λ*ˆ*SMB*

= 1 *SMB*

*T t*

*t*=1 *T*

*λ*ˆ = 1 Σ *HML .* (1.7)

*HML*

*T*

*t*=1

*t*

* To calculate the variance-covariance matrix of the risk premia estima- tors,

*V*ˆ*ar*(***λ***ˆ

) = 1 **Ω**ˆ

*K T K*

*T*

= (***Z****Kt*

Σ 1

*T* 2

*t*=1

*T*

***Z****Kt*

Σ1*−*

*T*

*t*=1

)(***Z****Kt*

*T*

***Z****Kt*

Σ1*−*

*T*

*t*=1

)*j.*

You need to calculate factor realizations ***Z****Kt*.

(1.8)

**Ω**ˆ *K* , which is the sample covariance matrix of

*s*ˆ*f*1*f*1

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**Ω**ˆ =

*K* *s*ˆ*f*1*f*2

*s*ˆ*f*1*f*3

*s*ˆ*f*1*f*2 *s*ˆ*f*2*f*2 *s*ˆ*f*2*f*3

*s*ˆ*f*1*f*3

(1.9)



*s*ˆ*f*2*f*3 

*s*ˆ*f*3*f*3

The diagonal entries of **Ω**ˆ *K* are the factor sample variances and the o - diagonal entries are the sample covariances between the factors.

To calculate the sample variance of each factor, use function VAR.S(). To calculate sample covariance between two factors, use function COVARIANCE.S().

Then, calculate *V ar*(***λ***ˆ*K* ) in equation [(1.8).](#_bookmark0)

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Finally, calculate the test statistics for each factor *k*:

*λ*ˆ*k*

*φk* = ˆ 1 *∼ N* (0*,* 1)*,* (1.10)

*V*ˆ*ar*[*λk*] 2

where *V ar*(*λ*ˆ*k*) is the element (*k, k*) in the matrix *V ar*[***λ***ˆ*K* ].

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1. Use the Likelihood ratio approach to test the model with three macroeco- nomic factors without the existence of risk-free asset. Calculate the risk premia of the factors.
   * Unrestricted model

***R****t* = ***α*** + ***β****IP IPt* + ***β****TSTSt* + ***β****OP OPt* + ***s****t* (1.11) E[***s****t*] = 0

E[***s****t****s****jt*] = **Σ**

***R****t* is an *N ×* 1 vector of realized returns for *N* stocks. *IP* is industrial production growth, *TS* is term structure spread, *OP* is oil price return.

The estimation steps are exactly the same as that for the unrestricted Fama-French three factor model, except that the dependent variables are realized returns, and the factors are the three macroeconomic variables.

* + Restricted model

***R****t* = *γ*0 + ***β****IP* (*IPt − γ*11) + ***β****TS*(*TSt − γ*12) + ***β****OP* (*OPt − γ*13) + ***s****t*

(1.12)

= ***ι****γ*0 *− βIP γ*11 *− βTSγ*12 *− βOP γ*13 + ***β****IP IPt* + ***β****TSTSt* + ***β****OP OPt* + ***s****t*

(1.13)

*γ*0 is the zero-beta portfolio return. *γ*11, *γ*12 and *γ*13 are, respectively, the risk free part of factor *IP* , *TS*, and *OP* . *γ*:s are parameters to be estimated. *H*0 : ***α*** = ***ι****γ*0 *− βIP γ*11 *− βTSγ*12 *− βOP γ*13; *H*1 : ***α*** *ƒ*= ***ι****γ*0 *− βIP γ*11 *− βTSγ*12 *− βOP γ*13*.*

1. Use realized returns from the data.
2. De ning initial values for the parameters. Since *N* = 5 *K* = 3, you need 15 initial values for *β*:s. One initial value for *γ*0 and three initial values for *γ*1*k*. Initial values are given in Lab3.xlxs.
3. 5 new columns for the residuals by referring to the initial values as

*eit* = *Rit−γ*0*−βi,IP* (*IPt−γ*11)*−βi,TS*(*TSt−γ*12)*−βi,OP* (*OPt−γ*13)*, i* = 1*,* 2*, . . .* 5

(1.14)

1. The rest of the estimation steps are the same as for other models above.
   * The null hypothesis can be tested using the likelihood ratio:

*LR* = *−*2(ln *L∗ −* ln *L*) *∼ χ*2

*N−K−*1

(1.15)

Use the function "CHIDIST(*LR*, *N −K−* 1)" to compute the p-value for the test statistics.

* + The factor risk premia are calculated as

*T*

*λ*ˆ = 1 Σ *IP*

*− γ*ˆ

(1.16)

*IP T*

1

*T*

Σ

*t* 11

*λ*ˆ*T S*

= 1 *TS*

*T t*

1

*T*

Σ

*− γ*ˆ12

*λ*ˆ*OP*

= 1 *OP*

*T t*

1

*− γ*ˆ13